LESSON

1.1

Building Blocks of Geometry

Nature's Great Book is written in mathematical symbols.

GALILEO GALILEI

Three building blocks of geometry are points, lines, and planes. A **point** is the most basic building block of geometry. It has no size. It has only location. You represent a point with a dot, and you name it with a capital letter. The point shown below is called P.

Mathematical model of a point

A tiny seed is a physical model of a point.

A **line** is a straight, continuous arrangement of infinitely many points. It has infinite length but no thickness. It extends forever in two directions. You name a line by giving the letter names of any two points on the line and by placing the line symbol above the letters, for example, $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$.

Mathematical model of a line

A piece of spaghetti is a physical model of a line. A line, however, is longer, straighter, and thinner than any piece of spaghetti ever made.

A **plane** has length and width but no thickness. It is like a flat surface that extends infinitely along its length and width. You represent a plane with a four-sided figure, like a tilted piece of paper, drawn in perspective. Of course, this actually illustrates only part of a plane. You name a plane with a script capital letter, such as $\mathcal{P}$.

Mathematical model of a plane

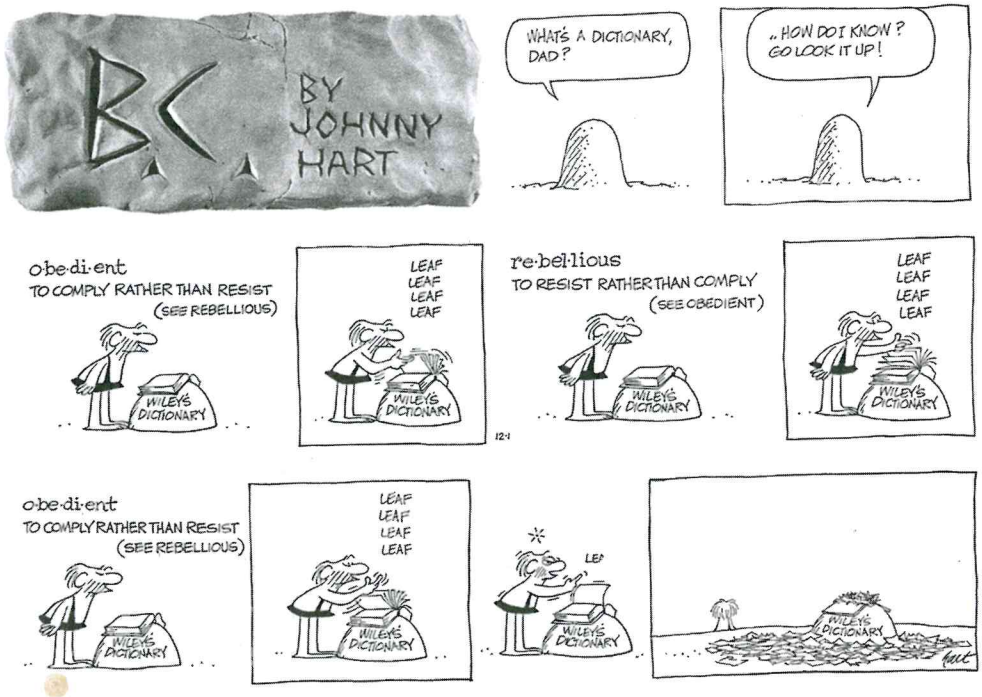
A flat piece of rolled-out dough is a model of a plane, but a plane is broader, wider, and thinner than any piece of dough you could roll.

Investigation
Mathematical Models

Step 1 | Identify examples of points, lines, and planes in these pictures.

Step 2 | Explain in your own words what point, line, and plane mean.

It can be difficult to explain what points, lines, and planes are. Yet, you probably recognized several models of each in the investigation. Early mathematicians tried to define these terms.



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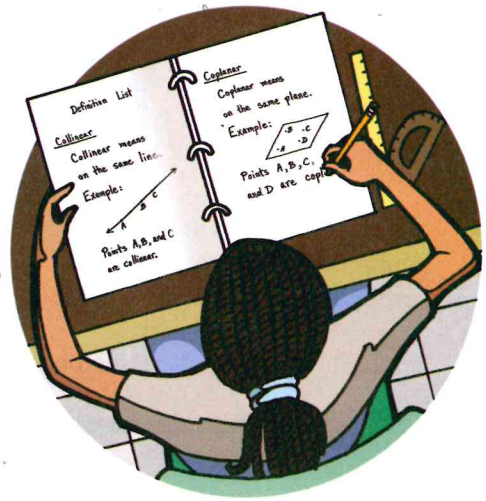
The ancient Greeks said, "A point is that which has no part. A line is breadthless length." The Mohist philosophers of ancient China said, "The line is divided into parts, and that part which has no remaining part is a point." Those definitions don't help much, do they?

A **definition** is a statement that clarifies or explains the meaning of a word or a phrase. However, it is impossible to define point, line, and plane without using words or phrases that themselves need definition. So these terms remain undefined. Yet, they are the basis for all of geometry.

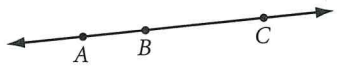
Using the undefined terms *point, line,* and *plane,* you can define all other geometry terms and geometric figures. Many are defined in this book, and others will be defined by you and your classmates.

Keep a definition list in your notebook, and each time you encounter new geometry vocabulary, add the term to your list. Illustrate each definition with a simple sketch.

Here are your first definitions. Begin your list and draw sketches for all definitions.

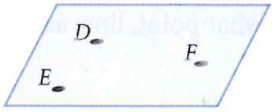


Collinear means on the same line.



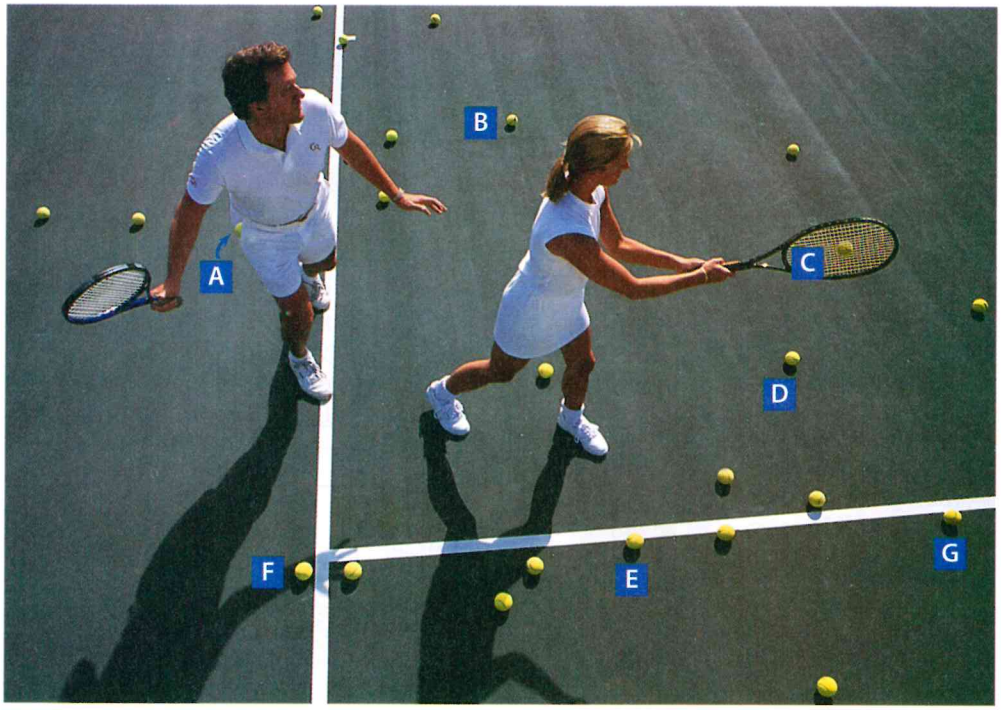
Points *A, B,* and *C* are collinear.

Coplanar means on the same plane.

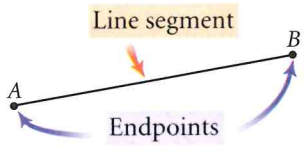


Points *D, E,* and *F* are coplanar.

Name three balls that are collinear. Name three balls that are coplanar but not collinear. Name four balls that are not coplanar.



A **line segment** consists of two points called the **endpoints** of the segment and all the points between them that are collinear with the two points.

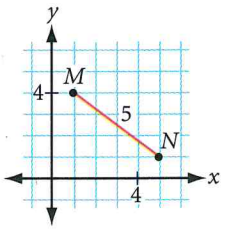


You can write line segment AB, using a segment symbol, as $\overline{AB}$ or $\overline{BA}$. There are two ways to write the length of a segment. You can write $AB = 2$ in., meaning the distance from A to B is 2 inches. You can also use an m for "measure" in front of the segment name, and write the distance as $m\overline{AB} = 2$ in. If no measurement units are used for the length of a segment, it is understood that the choice of units is not important, or is based on the length of the smallest square in the grid.

Figure A

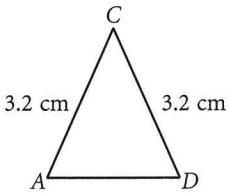
$AB = 2$ in., or $m\overline{AB} = 2$ in.

Figure B



$m\overline{MN} = 5$ units

Two segments are **congruent segments** if and only if they have the same measure or length. The symbol for congruence is $\cong$, and you say it as "is congruent to." You use the equals symbol, $=$, between equal numbers and the congruence symbol, $\cong$, between congruent figures.

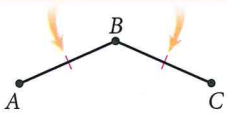


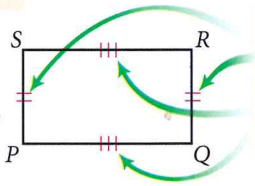
$AC = DC$

$\overline{AC} \cong \overline{DC}$

When drawing figures, you show congruent segments by making identical markings.

These single marks mean these two segments are congruent to each other.





These double marks mean that $\overline{SP} \cong \overline{RQ}$, and these triple marks mean that $\overline{PQ} \cong \overline{SR}$.

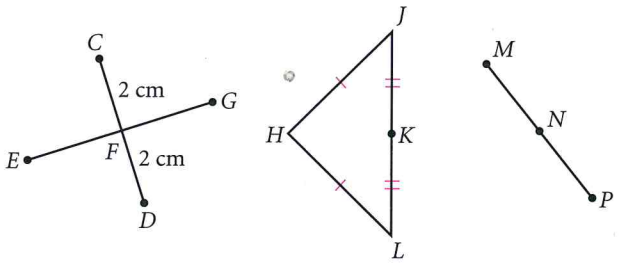
The **midpoint** of a segment is the point on the segment that is the same distance from both endpoints. The midpoint **bisects** the segment, or divides the segment into two congruent segments.

EXAMPLE

Study the diagrams below.

a. Name each midpoint and the segment it bisects.

b. Name all the congruent segments. Use the congruence symbol to write your answers.



▶ Solution

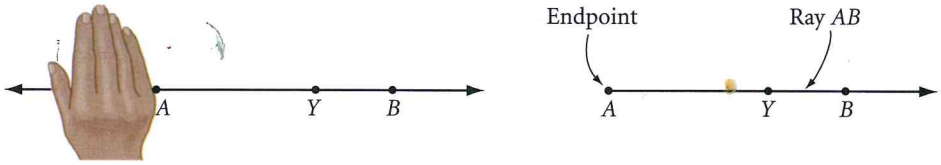
Look carefully at the markings and apply the midpoint definition.

a. $CF \cong FD$, so F is the midpoint of $\overline{CD}$; $\overline{JK} \cong \overline{KL}$, so K is the midpoint of $\overline{JL}$.

b. $\overline{CF} \cong \overline{FD}$, $\overline{HJ} \cong \overline{HL}$, and $\overline{JK} \cong \overline{KL}$.

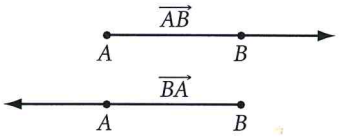
Even though $\overline{EF}$ and $\overline{FG}$ appear to have the same length, you cannot assume they are congruent without the markings. The same is true for $\overline{MN}$ and $\overline{NP}$.

Ray AB is the part of $\overleftrightarrow{AB}$ that contains point A and all the points on $\overleftrightarrow{AB}$ that are on the same side of point A as point B. Imagine cutting off all the points to the left of point A.



In the figure above, $\overrightarrow{AY}$ and $\overrightarrow{AB}$ are two ways to name the same ray. Note that $\overrightarrow{AB}$ is not the same as $\overrightarrow{BA}$!

A ray begins at a point and extends infinitely in one direction. You need two letters to name a ray. The first letter is the endpoint of the ray, and the second letter is any other point that the ray passes through.



Physical model of a ray: beams of light