LESSON 1.3

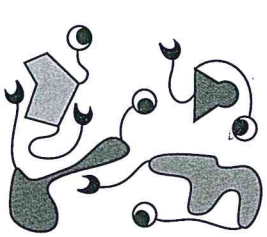
What's a Widget?

"When I use a word," Humpty replied in a scornful tone, "it means just what I choose it to mean—neither more nor less." "The question is," said Alice, "whether you can make a word mean so many different things."

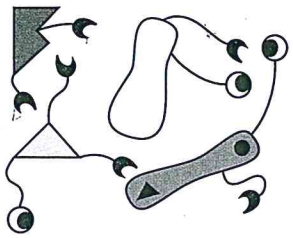
LEWIS CARROLL

Good definitions are very important in geometry. In this lesson you will write your own geometry definitions.

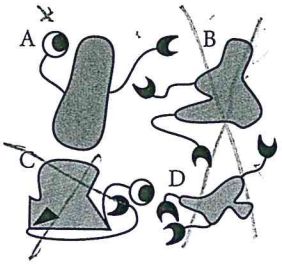
Which creatures in the last group are Widgets?



Widgets

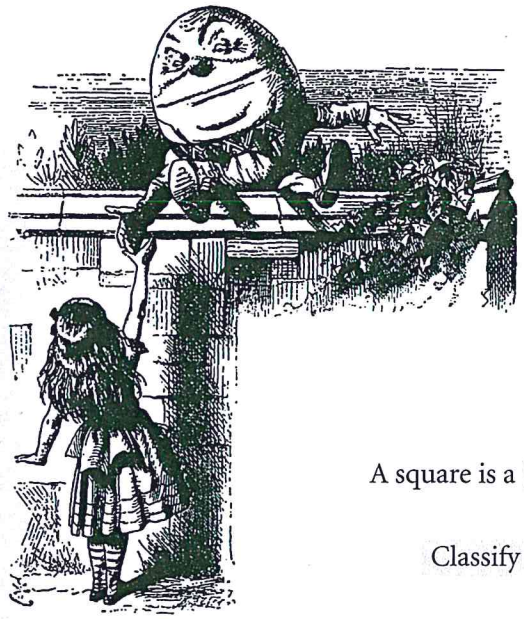


Not Widgets



Who are Widgets?

You might have asked yourself, "What things do all the Widgets have in common, and what things do Widgets have that others do not have?" In other words, what characteristics make a Widget a Widget? They all have colorful bodies with nothing else inside; two tails—one like a crescent moon, the other like an eyeball.



By observing what a Widget is and what a Widget isn't, you identified the characteristics that distinguish a Widget from a non-Widget. Based on these characteristics, you should have selected A as the only Widget in the last group. This same process can help you write good definitions of geometric figures.

This statement defines a protractor: "A protractor is a geometry tool used to measure angles." First, you classify what it is (a geometry tool), then you say how it differs from other geometry tools (it is the one you use to measure angles). What should go in the blanks to define a square?

A square is a _____ that _____.

Classify it. What is it? How does it differ from others?

Once you've written a definition, you should test it. To do this, you look for a **counterexample.** That is, try to create a figure that fits your definition but *isn't* what you're trying to define. If you can come up with a counterexample for your definition, you don't have a good definition.

EXAMPLE A

Everyone knows, "A square is a figure with four equal sides." What's wrong with this definition?

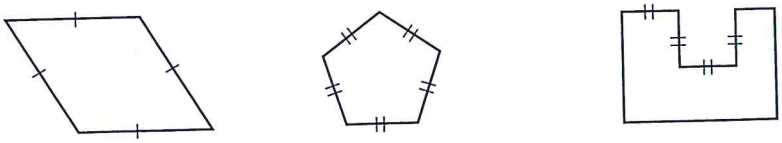
a. Sketch a counterexample. (You can probably find more than one!)

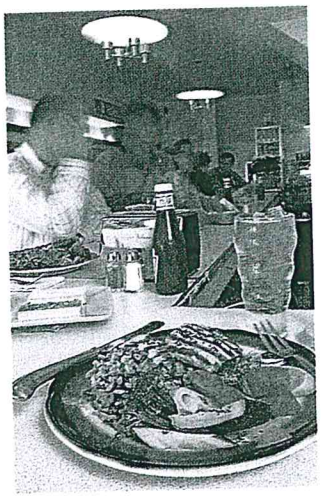
b. Write a better definition for a square.

▸ Solution

You probably noticed that "figure" is not specific enough to classify a square, and that "four equal sides" does not specify how it differs from the first counterexample shown below.

a. Three counterexamples are shown here, and you may have found others, too.





A restaurant counter example

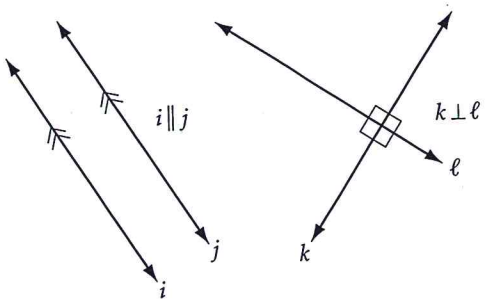
b. One better definition is "A square is a 4-sided figure that has all sides congruent and all angles measuring 90 degrees."

Beginning Steps to Creating a Good Definition

1. **Classify** your term. What is it? ("A square is a 4-sided figure . . .")
2. **Differentiate** your term. How does it differ from others in that class? (". . . that has four congruent sides and four right angles.")
3. **Test** your definition by looking for a counterexample.

Ready to write a couple of definitions? First, here are two more types of markings that are very important in geometry.

The same number of arrow marks indicates that lines are parallel. The symbol $\parallel$ means "is parallel to." A small square in the corner of an angle indicates that it measures 90°. The symbol $\perp$ means "is perpendicular to."



EXAMPLE B

Define these terms:

a. Parallel lines

b. Perpendicular lines

▸ Solution

Following these steps, classify and differentiate each term.

Classify. Differentiate.

a. Parallel lines are lines in the same plane that never meet.

b. Perpendicular lines are lines that meet at 90° angles.

Why do you need to say "in the same plane" for parallel lines but not for perpendicular lines? Sketch or demonstrate a counterexample to show the following definition is incomplete: "Parallel lines are lines that never meet." (Two lines that do not intersect and are noncoplanar are **skew** lines.)

Investigation
Defining Angles

Here are some examples and non-examples of special types of angles.

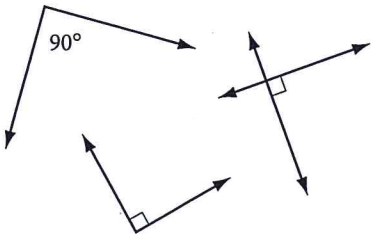
Step 1 Write a definition for each boldfaced term. Make sure your definitions highlight important differences.

Step 2 Trade definitions and test each other's definitions by looking for counterexamples.

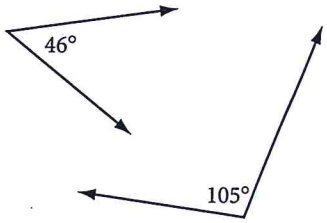
Step 3 If another group member finds a counterexample to one of your definitions, write a better definition. As a group, decide on the best definition for each term.

Step 4 As a class, agree on common definitions. Add these to your notebook. Draw and label a picture to illustrate each definition.

Right Angle



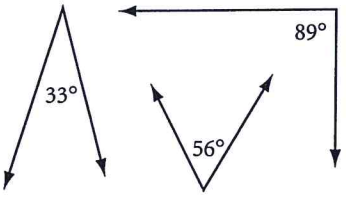
Right angles



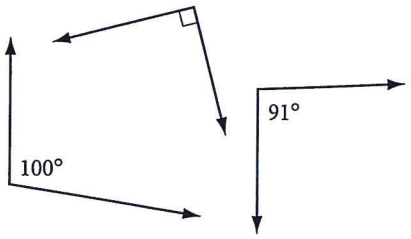
Not right angles

Notice the many congruent angles in this Navajo transitional Wedgeweave blanket. Are they right, acute, or obtuse angles?

Acute Angle

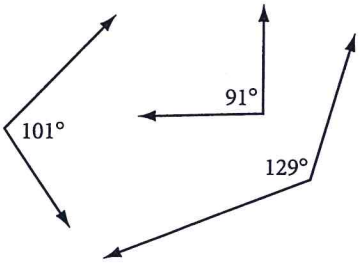


Acute angles

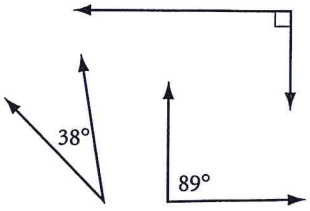


Not acute angles

Obtuse Angle

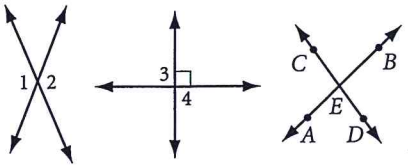


Obtuse angles



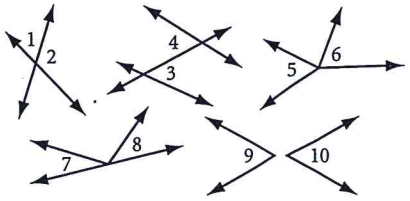
Not obtuse angles

Pair of Vertical Angles



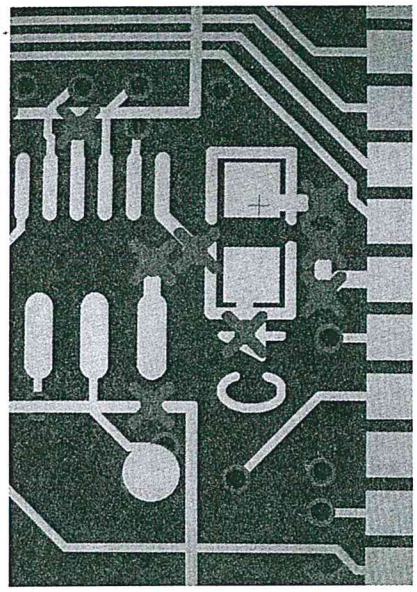
Pairs of vertical angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle AED$ and $\angle BEC$
$\angle AEC$ and $\angle DEB$



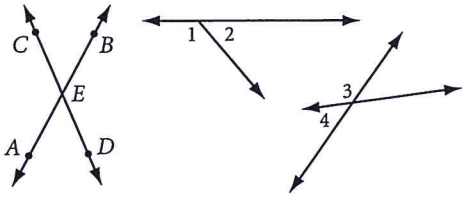
Not pairs of vertical angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle 5$ and $\angle 6$
$\angle 7$ and $\angle 8$
$\angle 9$ and $\angle 10$



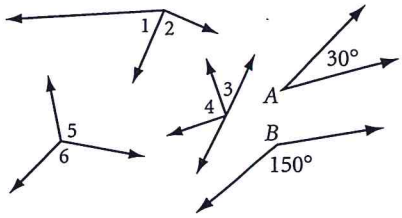
What types of angles or angle pairs do you see in this magnified view of a computer chip?

Linear Pair of Angles



Linear pairs of angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle AED$ and $\angle AEC$
$\angle BED$ and $\angle DEA$

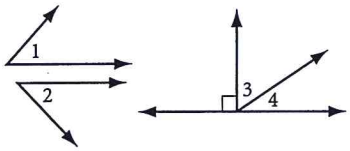


Not linear pairs of angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle 5$ and $\angle 6$
$\angle A$ and $\angle B$

Pair of Complementary Angles

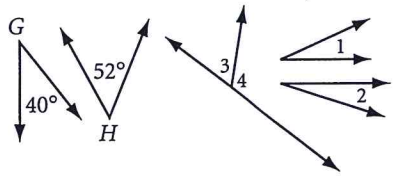
$m\angle 1 + m\angle 2 = 90°$



Pairs of complementary angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$

$m\angle 1 + m\angle 2 \neq 90°$

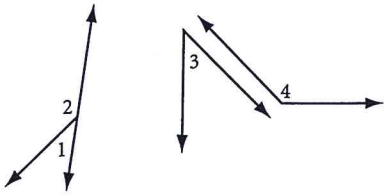


Not pairs of complementary angles:

$\angle G$ and $\angle H$ $\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$

Pair of Supplementary Angles

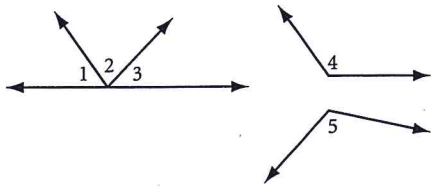
$m\angle 3 + m\angle 4 = 180°$



Pairs of supplementary angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$

$m\angle 4 + m\angle 5 > 180°$



Not pairs of supplementary angles:

$\angle 1$, $\angle 2$, and $\angle 3$
$\angle 4$ and $\angle 5$